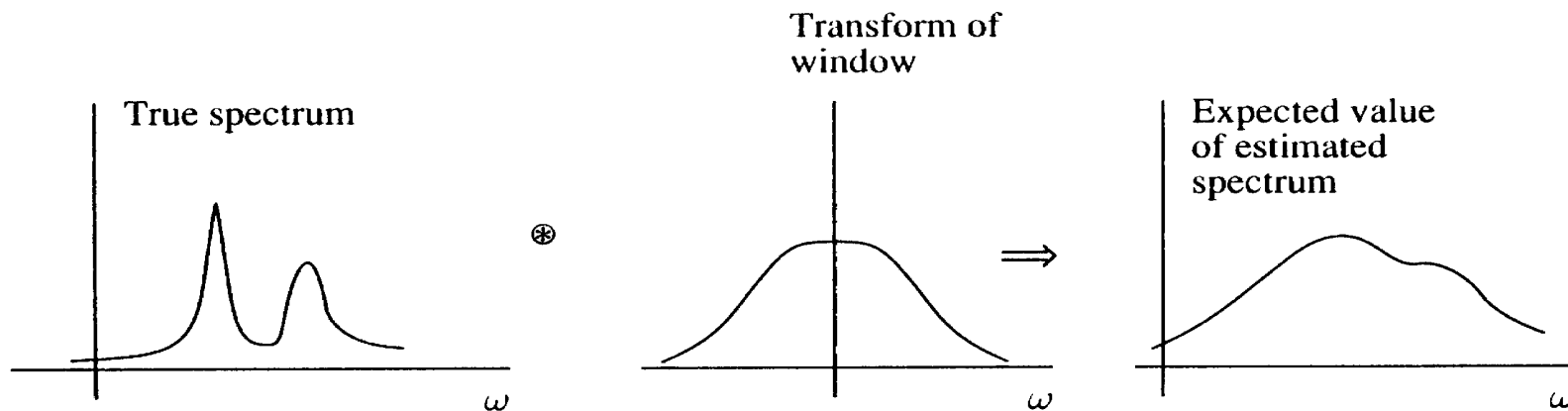


MODERN SPECTRUM ANALYSIS

- Methods based on linear models
 - AR
 - MA
 - ARMA
- AR method and Maximum Entropy
- “Maximum Likelihood” method

LIMITATIONS OF CLASSICAL METHODS

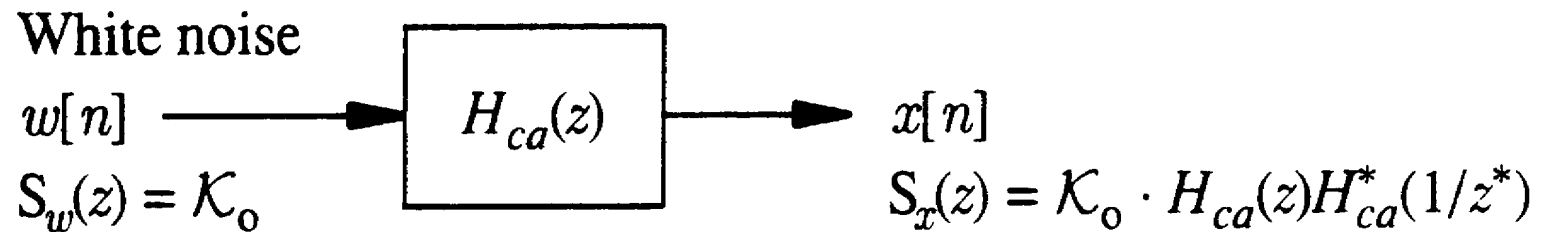
- Classical methods are limited in resolution by the data length.



- Methods based on a model for the process can overcome this limitation.

SPECTRAL ESTIMATION USING A LINEAR MODEL

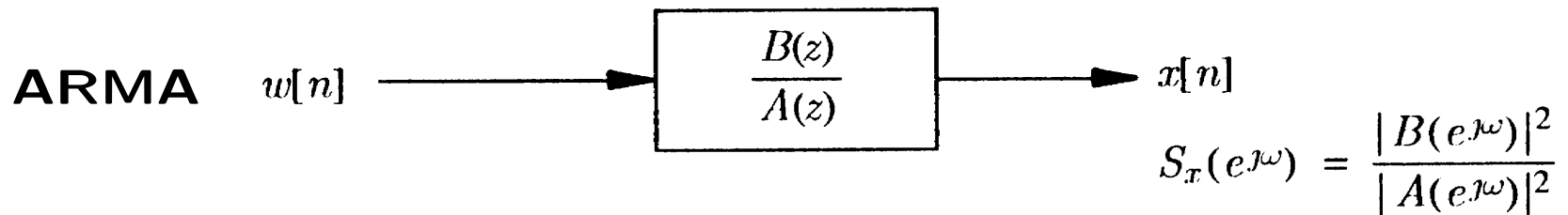
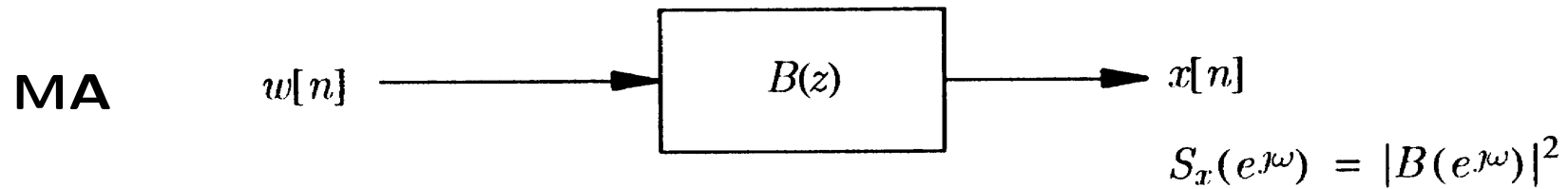
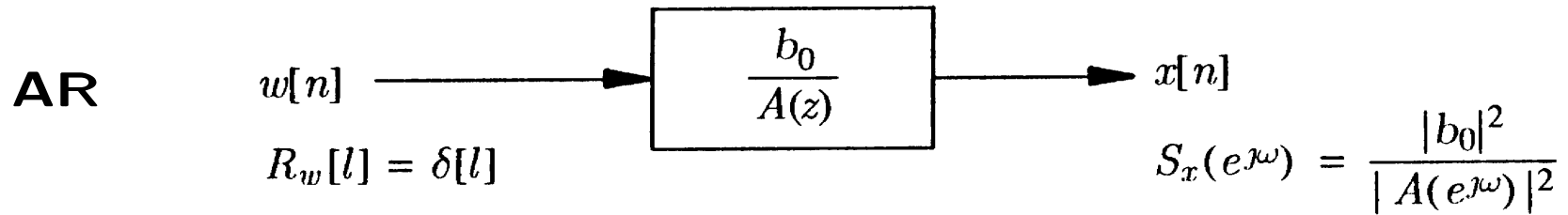
MODEL FOR THE PROCESS



SPECTRAL ESTIMATE

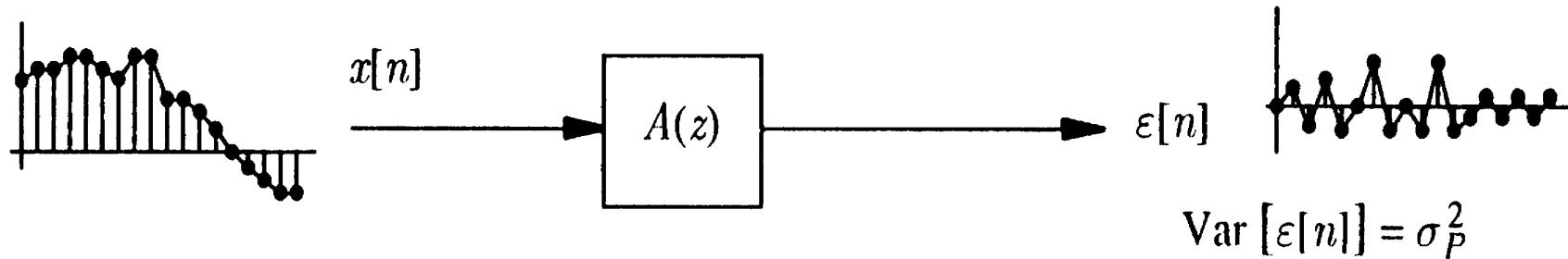
$$\hat{S}(e^{j\omega}) = \mathcal{K}_o |H_{ca}(e^{j\omega})|^2$$

TYPES OF LINEAR MODELS

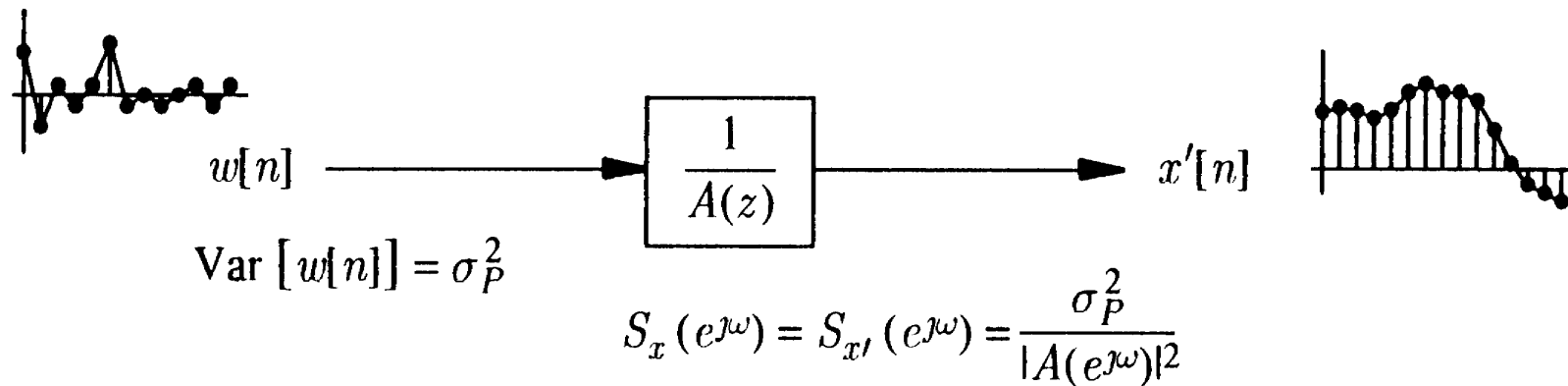


SPECTRUM ESTIMATION BY AR MODELING

LINEAR PREDICTION



AR MODELING



PROPERTIES OF THE AR MODEL

CORRELATION MATCHING

$$R_{x'}[l] = R_x[l] ; \quad l = 0, \pm 1, \pm 2, \dots, \pm P$$

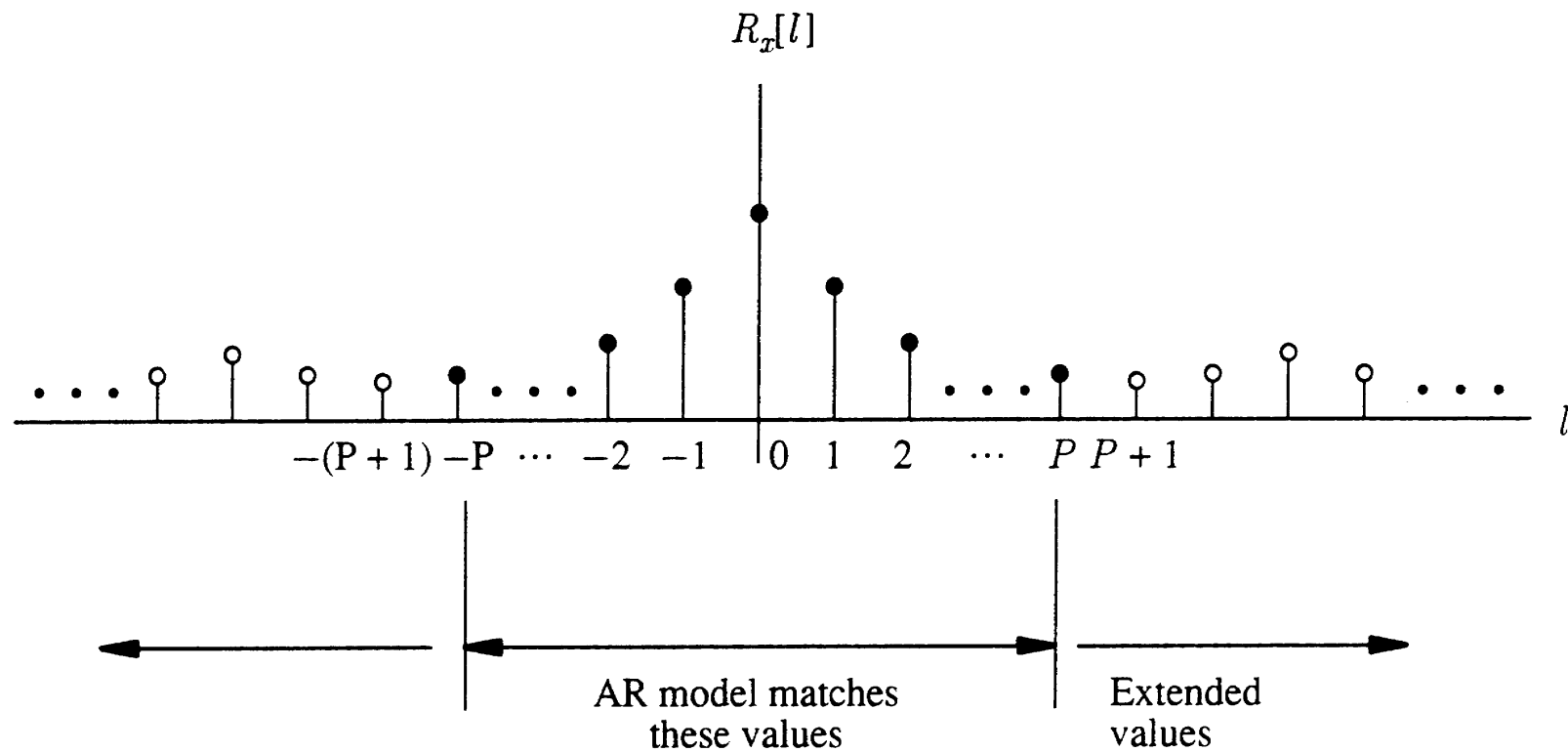
CORRELATION EXTENSION

$$R_{x'}[l] + a_1 R_{x'}[l - 1] + a_2 R_{x'}[l - 2] + \dots + a_P R_{x'}[l - P] = \underbrace{R_{wx'}[l]}_{0 \text{ for } l > 0}$$

\Rightarrow

$$R_{x'}[l] = -a_1 R_{x'}[l - 1] - a_2 R_{x'}[l - 2] - \dots - a_P R_{x'}[l - P], \quad l > 0$$

MATCHING AND EXTENSION OF THE CORRELATION FUNCTION



MAXIMUM ENTROPY PROPERTY

- The AR model of order P matches the correlation function up to lag P .
- The AR model extends the correlation function in a way to maximize entropy of the resulting process.

In other words . . .

- Of all processes that could match and extend the given correlation function, the AR process is the process with maximum entropy.

“MAXIMUM ENTROPY” SPECTRUM ESTIMATION

- “Maximum Entropy” is the name given by Burg to his method of spectrum estimation.
- Theoretically, any AR spectral estimate is a maximum entropy spectral estimate.
- In practice, the term is reserved to mean an AR estimate where the model parameters are computed using Burg’s method.

COMPUTATION OF SPECTRAL ESTIMATES

- Efficient model-based spectral estimates can be computed with the FFT.

- Note that

$$\hat{S}(e^{j\omega}) = \left| \frac{B(e^{j\omega})}{A(e^{j\omega})} \right|^2 = \frac{|\text{FT of sequence } \{b_n\}|^2}{|\text{FT of sequence } \{a_n\}|^2}$$

- The FT can be replaced by the DFT computed with an FFT program; for purposes of plotting, the DFT order is chosen to give a smooth plot.